Transition from Classical into Quantum Theory for Calculating the Stopping Power of Charged Particles

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Abstract

In this search, we described the most important corrections to standard energy loss formula of charged particles and discussed this formalism with a few remarks by the implication of the inverse – Bloch correction to the stopping power calculations. The inverse Bloch correction extends the range of validity of classical Bohr and quantum Bethe regime. Bohr model operates with a charged particles penetrating in matter as a number of interactions with a harmonically bound classical target electrons but in Bethe theory, all the calculations of energy loss of fast charged particles are based on the first Born approximation which the entire physical system is considered quantized. All the results are got by programming the equations in matlab program.

1.Introduction

The interaction of charged particles with matter has been a subject of great importance for the fundamental interaction processes, as well as for the practical application [1]. There are many phenomena which may be included in a complete description of energy loss and they may be described in many applications over many decades [2].

In the interaction between charged particles and matter, the stopping power is defined by $S(E)$. If the radiation energy loss is unimportant and neglected, it is also called linear energy transfer and proportional to v^2. The less interaction it has with the particles in the matter, the faster the particle is [3]. Because the penetrating particles generate excitation and ionization, the ionization level increases along and in the vicinity of the penetrating path. This is especially when the primary particle has slowed down to KeV, and hundreds, or even tens of eV in energy. The ion – electron pairs may recombine, albeit very slowly, in some dielectrics, depending on the conductivity of the materials. The increase in the net charge, however, is independent of the amount of ionization [3].

One can easily estimate the stopping power over a very broad range of ion – target combinations as well as beam velocities [4].

Bohr approach, which is dependent on the impact parameter between the particle trajectory and the target nucleus, and Bethe approach which depends on momentum transfer from the particle to the target electrons. Bethe's approach was necessary since quantum mechanics prohibits a particle with a well defined momentum having a spatially localized position [5]. Bohr's concept of an impact parameter was defined in 1913, before quantum mechanics was developed [5].

The stopping of swift charged particles impinging on matter, for large projectile velocities, determined by the mean excitation energy I of the target material through the logarithmic term in the Bethe formula. The electronic shell corrections to the Bethe formula are important, however for the lower velocities most experiments are carried out [6].

Bloch evaluated the differences between the classical (Bohr) and quantum – mechanical (Bethe) approaches for particles with velocities much larger than the target electrons. Bloch showed that Bohr's approach was valid also in the quantum mechanics of a bound electron if the energy transferred was assumed to be the mean energy loss, summed over all possible atomic transitions[5].

2.The Theoretical side

2.1 Bohr Theory

The fundamental process in the Bohr theory is the interaction of a point charge $Z_1 e$ in uniform motion with an electron at rest, bound harmonically with a resonance frequency ω. The interaction is easily seen to be characterized by the dimensionless parameters, $\zeta = \omega p/v$ for the impact parameter p and $\xi = mv^3/Z_1 e^2\omega$ for the projectile speed v [7]. A fundamental aspect of the Bohr theory is the splitting into two regimes for small and large impact parameters [8].

Close interactions are treated as free – Coulomb and taken to follow Rutherford's for the energy loss T versus impact parameter [8].The stopping cross section for close interaction S_{close} [9]

$$S_{close} = \frac{2\pi Z_1^2 e^2}{mv^2} \ln\left[1 + \left(\frac{2p_0}{b}\right)^2\right] \qquad (1)$$

and the stopping number for close interaction L_{close}

$$L_{close} = \frac{1}{2} \ln\left[1 + \left(\frac{2p_0}{b}\right)^2\right] \qquad (2)$$

Distant interactions are described as excitation of harmonic oscillators by a time varying electronic field in which the binding of target electrons is taken into account and enters through a classical resonance frequency ω [8].The stopping cross section for distant interaction $S_{distant}$ [9]

$$S_{distant} = \frac{4\pi Z_1^2 e^2}{mv^2} \frac{\omega p_0}{v} K_0\left(\frac{\omega p_0}{v}\right) K_1\left(\frac{\omega p_0}{v}\right) \qquad (3)$$

and the stopping number for distant interaction $L_{distant}$

$$L_{distant} = \frac{\omega p_0}{v} K_0\left(\frac{\omega p_0}{v}\right) K_1\left(\frac{\omega p_0}{v}\right) \quad (4)$$

the summation of eqs.(2 and 4) leads to the total stopping number [9]

$$L_{total} = L_{close} + L_{distant} \quad (5)$$

$$L_{total} = \frac{1}{2}\ln\left[1 + \left(\frac{2p_0}{b}\right)^2\right] + \frac{\omega p_0}{v} K_0\left(\frac{\omega p_0}{v}\right) K_1\left(\frac{\omega p_0}{v}\right) \quad (6)$$

where $Z_1$is the atomic number of projectile, e is the charge of an electron, m is the mass of an electron, p_0 is the critical impact parameter, K_0 and K_1 are modified Bessel function in standard notation and $b = 2Z_1e^2/mv^2$ is the collision diameter. Bohr's evaluation is based on the recognition that, at high speed where $b \ll v/\omega$, a value of p_0 may be found such that $b \ll p_0 \ll v/\omega$ [9].

$$L_{close} = \ln\left(\frac{2p_0}{b}\right) \quad (7)$$

$$L_{distant} = \ln\left(\frac{2ve^{-\gamma}}{\omega\, p_0}\right) \quad (8)$$

Where γ =0.5772 is the Euler's constant. The summation of eqs.(7 and 8) leads to the Bohr stopping number [9]

$$L_{Bohr} = \ln\left(\frac{2p_0}{b}\right) + \ln\left(\frac{2ve^{-\gamma}}{\omega\, p_0}\right) = \ln\left(\frac{2p_0}{b}\frac{2ve^{-\gamma}}{\omega p_0}\right) \quad (9)$$

by substitution the value of b and with $C = 2e^{-\gamma}$, the above equation becomes [9]

$$L_{Bohr} = \ln\left(\frac{Cmv^3}{Z_1e^2\omega}\right) = \ln(\mathrm{C}\xi) \quad (10)$$

where $\xi = \frac{mv^3}{Z_1e^2\omega}$ is the Bohr parameter and $C = 1.1229$, eq.(10) is the Bohr's original formula for the stopping number of particle which may take over – at least as a feasible starting point – in the velocity range where this has to be expected from Bohr's kappa criterion, $K = \frac{2Z_1v_0}{v}$ >1 [8] in order to evaluate p_0,$L_{close} = L_{distant}$ at p_0 and from eqs.(7 and 8) [9]

$$\ln\left(\frac{2p_0}{b}\right) = \ln\left(\frac{2ve^{-\gamma}}{\omega\, p_0}\right) = \frac{2p_0}{b} = \frac{2ve^{-\gamma}}{\omega\, p_0} \quad (11)$$

by substitution the value of b and with $C = 2e^{-\gamma}$, the critical impact parameter $p_0$is

$$p_0 = \left(\frac{CZ_1e^2}{\omega\, m\, v}\right)^{1/2} \quad (12)$$

with $x = \frac{wp_0}{v}$, after substitution the critical impact parameter p_0, x becomes

$$x = \frac{\omega}{v}\left(\frac{CZ_1e^2}{\omega\, m\, v}\right)^{1/2} = \left(\frac{CZ_1e^2\omega}{m\, v^3}\right)^{1/2} \quad (13)$$

$$x = \left(\frac{C}{\xi}\right)^{1/2} \qquad \text{with } \frac{1}{\xi} = \frac{Z_1e^2\omega}{m\, v^3} \quad (14)$$

also the value of $\frac{2p_0}{b}$ may be found by substituting p_0 eq.(12) and b in it

$$\frac{2p_0}{b} = \frac{mv^2}{Z_1e^2}\left(\frac{CZ_1e^2}{\omega\, m\, v}\right)^{1/2} = \left(\frac{C\, m\, v^3}{Z_1e^2\omega}\right)^{1/2}$$

(15)

$$\left(\frac{2p_0}{b}\right)^2 = C\xi \qquad \text{with } \xi = \frac{mv^3}{Z_1 e^2 \omega} \tag{16}$$

in order to evaluate the total stopping number of close and distant interactions, eqs.(14 and 16) must be substituted into eq.(6).

For a point charge and small impact parameter, the energy loss is [10]

$$T(p) \sim \frac{2Z_1^2 e^4}{mv^2 p^2} \tag{17}$$

at $p = 0$, the energy loss becomes infinite. However, the replacement $p^2 \to p^2 + p_0^2$ with the critical impact parameter $p_0 = Z_1 e^2 / mv^2$, turns eq.(17) into Rutherford's law which is suitable for close collisions. It is possible to make a substitution for all impact parameters, but this has a little effect at large impact parameter p. The result of the integration of the stopping cross section $S = \int T(p) 2\pi\, p\, dp$ in closed form is [10]

$$L = \frac{\omega p_0}{v} K_0\left(\frac{\omega p_0}{v}\right) K_1\left(\frac{\omega p_0}{v}\right) \tag{18}$$

with the critical impact parameter $p_0 = \frac{Z_1 e^2}{mv^2}$, the value of $\frac{\omega p_0}{v}$ becomes

$$\frac{\omega p_0}{v} = \frac{\omega}{v}\left(\frac{Z_1 e^2}{mv^2}\right) = \frac{Z_1 e^2 \omega}{mv^3} = \frac{1}{\xi} = \xi^{-1} \qquad \text{with } \frac{1}{\xi} = \frac{Z_1 e^2 \omega}{m\, v^3} \tag{19}$$

hence, the equation (18) is

$$L = \xi^{-1} K_0(\xi^{-1}) K_1(\xi^{-1}) \tag{20}$$

the above equation represents a modification of Bohr's stopping number that extends the meaningful behavior down to lower projectile speeds than the original equation, eq.(10), and is readily reduced to Bohr's original formula eq.(10) at large Bohr parameter (ξ) [10].

2.2 Bethe Theory

The stopping power of light charged particles moving with velocity v in penetrating matter is proportional to Z_1^2, the square of its charge. The famous Bethe formula approximates the stopping number by [11]

$$L_{Bethe} = \ln \frac{2mv^2}{I} \tag{21}$$

where I is the mean excitation energy, eq.(21) valids at high speed but becomes unphysical and breaks down at low speed when $2mv^2 < I$ [11]. The validity of Bethe theory may be expressed in terms of Bohr Kappa criterion when it smaller than 1, i.e. $K = \frac{2Z_1 v_0}{v} < 1$, $v_0 = \frac{e^2}{\hbar}$ is the Bohr velocity [7].

for heavy ions the Bethe limit is reached at higher projectile speed than for light ions [10]

$$L = \beta^2 \ln \frac{2mv^2}{I} + (1 - \beta^2) \ln \frac{2mav}{\hbar} - \frac{1}{2}(1 - \beta)^2 \tag{22}$$

Where a is the screening radius, for a point charge ($\beta = 1$) the above equation reduces to the Bethe formula eq.(21) but for a neutral projectile ($\beta = 0$) it reduces to [10]

$$L = \ln \frac{2mav}{\hbar} - \frac{1}{2} \tag{23}$$

eq.(22) is of importance mainly for the identification of the upper limit of validity of the modified Bohr formula. Therefore, it is useful to write it down in terms of variables Bohr parameter ξ and velocity – independent parameter s, this yields [10]

$$L = \frac{1}{3}(1-\beta^2)\ln \xi + \left(\beta^2 - \frac{1}{2}\right)\ln s + \ln\left(2Z_1^{1/3}\right) + (1-\beta^2)\ln[0.885\, g(\beta)] - \frac{1}{2}\delta^2 \tag{24}$$

a clear dependence on the atomic number of projectile Z_1[10]. $\beta = \left(\frac{q_1}{Z_1}\right)$ is the charge fraction and s is a measure of the importance of screening at given ξ and β [10]

$$s = \left(\frac{Z_1 e^2/a_0}{\hbar\omega}\right)^{2/3} \tag{25}$$

a_0 being the Bohr radius. If the target is characterized by a single resonant frequency $\omega = I/\hbar$, and the Bloch's relation is

$$I = Z_2\, I_0 \tag{26}$$

Z_2 being the atomic number of target [12] and $I_0 \cong 10\, eV$ [13] is inserted for the mean excitation energy I, then eq.(25) is [10]

$$s = \left(\frac{Z_1}{Z_2}\right)^{2/3} \tag{27}$$

it is noted that the importance of screening at constant ξ and β increases with increasing $Z_1$and decreasing Z_2 and that s is equally sensitive to the target as to the projectile [10]. At the spectrum of resonant frequencies ω, the increase in s with decreasing ω explains the longer range of interactions with outer electrons, which are therefore more sensitive to projectile screening [10].

$$g(\beta) = \frac{a}{a_{TF}} \tag{28}$$

$g(\beta)$ is a dimensionless function of the charge fraction so that for a neutral projectile $g(0) = 1$ and $a_{TF} = \frac{0.8853 a_0}{Z_1^{1/3}}$ is the Thomas – Fermi radius [10]. $g(\beta)$ need numerical information and eq.(28) shows the dependence of the ionic screening radius on charge state [10]. Screening functions can be determined from Thomas – Fermi theory. $g(\beta)$ has been determined by matching the exponentially screened Coulomb potential to the charge distribution. Within the exactness of an exponential fit a Thomas – Fermi screening function, it seemed to adopt the expression [10].

$$g(\beta) = (1-\beta)^r = \delta^r \tag{29}$$

and by the connection of the two eqs.(28 and 29) leads to

$$\frac{a}{a_{TF}} = (1-\beta)^r = \delta^r \tag{30}$$

2. 3 Inverse Bloch Correction

The standard form of the Bloch terms reads [11]

$$L_{Bloch} = L_{Bethe} + \Delta L_{Bloch} \quad (31)$$

$$\Delta L_{Bloch} = -\gamma - R\psi\left(1 + i\frac{Z_1 v_0}{v}\right) \quad (32)$$

$$\Delta L_{Bloch} = \psi(1) - R\psi\left(1 + i\frac{Z_1 v_0}{v}\right) \quad (33)$$

where ΔL_{Bloch} is the Bloch correction, the Bloch formula reduces to Bohr formula $L_{Bloch} = L_{Bohr}$ at low projectile speed but at high projectile speed, ΔL_{Bloch} goes to zero and hence $L_{Bloch} = L_{Bethe}$, ψ is the logarithmic derivative of gamma function $[\psi(x) = \frac{d}{dx}\ln \Gamma(x)]$, and the numerical value of $\psi(1) = -0.5772 = -\gamma$ is the negative of Euler's constant (γ) [2] and R is the real part. By substitution eqs.(21 and 32) into eq.(31), one can get

$$L_{Bloch} = \ln\frac{2mv^2}{I} - \gamma - R\psi\left(1 + i\frac{Z_1 v_0}{v}\right) \quad (34)$$

eq.(31) for the Bloch stopping number L_{Bloch} may be written in an alternative way [11]

$$L_{Bloch} = L_{Bohr} + \Delta L_{invBloch} \quad (35)$$

where $\Delta L_{invBloch}$ is the inverse Bloch correction which may be got by substitution eqs.(10 and 34) into eq.(35) [11]

$$\Delta L_{invBloch} = \ln\frac{2mv^2}{I} - \gamma - R\psi\left(1 + i\frac{Z_1 v_0}{v}\right) - \ln\left(\frac{Cmv^3}{Z_1 e^2 \omega}\right) \quad (36)$$

$$\Delta L_{invBloch} = \ln\left(\frac{2mv^2}{I}\frac{Z_1 e^2 \omega}{Cmv^3}\right) + \ln e^{-\gamma} - R\psi\left(1 + i\frac{Z_1 v_0}{v}\right) \quad (37)$$

with $C = 2e^{-\gamma}$, $v_0 = e^2/\hbar$ is the Bohr velocity and $\omega = I/\hbar$

$$\Delta L_{invBloch} = \ln\left(\frac{Z_1 e^2}{\hbar\, v}\right) - R\psi\left(1 + i\frac{Z_1 e^2}{\hbar\, v}\right) \quad (38)$$

eq.(38) is the inverse Bloch correction (quantum correction) which ensures a proper approach to the Bethe formula at high velocities but it vanishes at low speed [11].

In the classical limit expressed by the Bohr Kappa criterion K, the inverse Bloch correction goes as [14]

$$\ln\left(\frac{Z_1 e^2}{\hbar\, v}\right) - R\psi\left(1 + i\frac{Z_1 e^2}{\hbar\, v}\right) \sim -\frac{1}{3K^2} - \frac{2}{15K^4} + \cdots \quad (39)$$

with $K = \frac{2\alpha}{\xi^{1/3}}$ and $\alpha = Z_1^{2/3}\left(\frac{e^2/a_0}{\hbar\, v}\right)^{1/3}$, where $a_0 = \frac{\hbar^2}{me^2}$ is the Bohr radius. In order to generate a modified Bloch formula, it must be appeared tempting to replace the Bohr logarithm by the total stopping number, eq.(6) which does not turn negative at low projectile speed [9].

2.4 Modified Inverse Bloch Correction

The inverse Bloch correction has been useful to calculate the stopping power outside the classical regime [15,16,17]. The problems occur at low projectile speed which the correction is insignificant (finite), approaches zero for $v \to 0$, and the stopping power is positive. The fact of the problems is that the Bethe logarithm becomes negative at low v, to avoided this problem by cutting the uncorrecting stopping number and using a modified inverse Bloch correction of Lindhard and Sorensen approximation to Bloch's formula [18].

$$L_{Bloch} = \ln \frac{2mv^2/\hbar\,\omega}{\sqrt{1+\left(\frac{e^{\gamma} Z_1 v_0}{v}\right)^2}} \quad (40)$$

With $\eta = \frac{Z_1 v_0}{v} = \frac{K}{2}$ is the Sommerfeld parameter, $\frac{2mv^2}{\hbar\,\omega} = 2\xi\,\eta$ and $C = 2e^{-\gamma}$, By substitution eqs.(10 and 40) into eq.(35), the inverse Bloch correction is

$$-\Delta L_{invBloch} = \ln C\xi - \ln\left(\frac{2mv^2/\hbar\,\omega}{\sqrt{1+(e^{\gamma}\eta)^2}}\right) \quad (41)$$

$$-\Delta L_{invBloch} = \ln \frac{2mv^2/\hbar\,\omega}{e^{\gamma}\eta} - \ln \frac{2mv^2/\hbar\,\omega}{\sqrt{1+(e^{\gamma}\eta)^2}}$$

$$-\Delta L_{invBloch} = \ln \sqrt{1+(e^{\gamma}\eta)^2} - \ln e^{\gamma}\eta = \ln \frac{\sqrt{1+(e^{\gamma}\eta)^2}}{e^{\gamma}\eta}$$

$$-\Delta L_{invBloch} = \ln \frac{e^{\gamma}\eta\,\sqrt{1+(1/e^{\gamma}\eta)^2}}{e^{\gamma}\eta} = \ln \sqrt{1+(1/e^{\gamma}\eta)^2}$$

$$-\Delta L_{invBloch} = \frac{1}{2}\ln(1+(1/e^{\gamma}\eta)^2) \quad (42)$$

it is seen that eq.(40) is reduced to the Bethe logarithms eq.(21) for $e^{\gamma}\eta \ll 1$, hence, $\sqrt{1+(e^{\gamma}\eta)^2} \cong \sqrt{1} = 1$

$$L_{Bloch} = \ln \frac{2mv^2/\hbar\,\omega}{1} = L_{Bethe}$$

but for $e^{\gamma}\eta \gg 1$, eq.(40) is reduced to the Bohr logarithms eq.(10), hence, $\sqrt{1+(e^{\gamma}\eta)^2} \cong \sqrt{(e^{\gamma}\eta)^2} = e^{\gamma}\eta$, with $\frac{2mv^2}{\hbar\,w} = 2\xi\,\eta$ and $e^{\gamma} = 2/C$

$$L_{Bloch} = \ln \frac{2mv^2/\hbar\,\omega}{e^{\gamma}\eta} = \ln C\xi = L_{Bohr}$$

The inverse Bloch correction is negative and causes the stopping number vanishes at some velocities. The fact of the problem create in the cutting of the Bohr stopping number at low v. Eq. (40) arises from the uncorrected Bohr formula by substituting [18]

$$\eta \to \sqrt{(\eta)^2 + e^{-2\gamma}} \quad (43)$$

A suitable formula for L_{Bloch} in the low velocity limit could be obtained by evaluating the stopping number from the total stopping number and fitting the result by the formula

$$L_{approx} = \ln(B + C\xi) - \ln B$$

$$L_{approx} = \ln\left(\frac{B+C\xi}{B}\right) = \ln\left(1+\frac{C\xi}{B}\right) \quad (44)$$

which has the correct form at high values of ξ, but it approaches zero at $v = 0$. By applying the same substitution for eq.(43) in eq.(44) yields the modified Bloch function [18]

$$L_{Bloch} = \ln\left(1 + \frac{2mv^2/\hbar\,\omega}{B\sqrt{1+(e^{\gamma}\eta)^2}}\right) \quad (45)$$

$B = 0.880$, by substitution eqs.(44 and 45) into eq.(35) to get the inverse Bloch correction

$$-\Delta L_{invBloch} = \ln(1 + \frac{\mathrm{C}\xi}{\mathrm{B}}) - \ln\left(1 + \frac{2mv^2/\hbar\,\omega}{B\sqrt{1+(e^{\gamma}\eta)^2}}\right) \quad (46)$$

in the Bethe limit, eq.(45) reduces to

$$L_{Bloch} = \ln\left(1 + \frac{2mv^2/\hbar\,\omega}{B}\right) \quad (47)$$

but in the Bohr limit, it reduced to eq.(44), with $\frac{2mv^2}{\hbar\,\omega} = 2\xi\,\eta$ and $e^{\gamma} = 2/C$

$$L_{Bloch} = \ln\left(1 + \frac{2C\xi\,\eta}{2B\eta}\right) = \ln\left(1 + \frac{C\xi}{B}\right) \quad (48)$$

the desired inverse Bloch correction is found from $\Delta L_{invBloch} \cong L_k - L|_{k=\infty}$[18]

$$-\Delta L_{invBloch} = -\frac{1+2\xi/\left(B\sqrt{C^2+(1/\eta)^2}\right)}{1+2\xi/(BC)} \quad (49)$$

3. Results and Discussion

Figure (1) shows a comparison between the results of stopping number evaluated from classical theory and obtained from Bohr formula (Bohr logarithm) the equation(10), modified Bohr formula the equation(20) and the total stopping number which included the contributions from close and distant collisions the equation(6) as a function of Bohr parameter (ξ) which means that the dependence of the stopping number on(ξ). It is seen that the Bohr logarithm turns negative at ($\xi = 1$) but then in the modified Bohr formula avoids the negative of stopping power without addition any of necessary low velocity corrections. There is an obvious difference between the curves from (eqs.6 and 20) at ($\xi \leq 5$) but this difference is significant at ($\xi \leq 2$). While the error edge in L for($\xi \leq 2$) appears larger than one might like it to be, Barkas corrections (neglected presently) may cause the errors. The two curves (eq.10 and 6) are concurrent at high ξ (high projectile speed) but differ considerably at ($\xi \leq 3$). It is also noted that in the low speed projectile, both types of interactions are controlled but the high speed projectile does not reproduce the commonly accepted equipartition between close and distant contributions to the stopping number. This feature is attributed to the exceed of the distant collision at high values of speed.

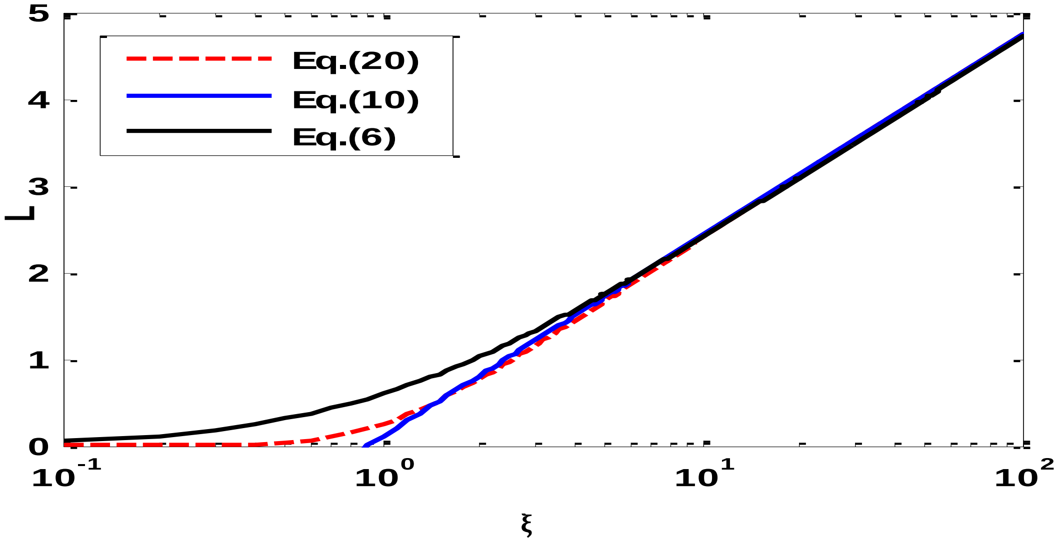


Fig.(1)Stopping number for a point charge evaluated from classical theory

Figure (2) shows the results of charge function $g(\beta)$ as a function of charge state β that are evaluated from eq.(30) which is extracted from Thomas – Fermi theory by matching to the exponentially screened Coulomb potential with different values of parameter $r = 0.5,\ 1,\ 1.5$. From the figure, it is noted that the dependence of screening radius on charge state and this influence is noticeable but it appears weaker than the effect of absolute value of screening which acts an inverse variation in velocity parameter s. The figure also shows that the influence of the parameter (r) which governs the dependence of the screening radius on charge state. At the same time we note that a value at $r = 1$ predicts the results with good accuracy.

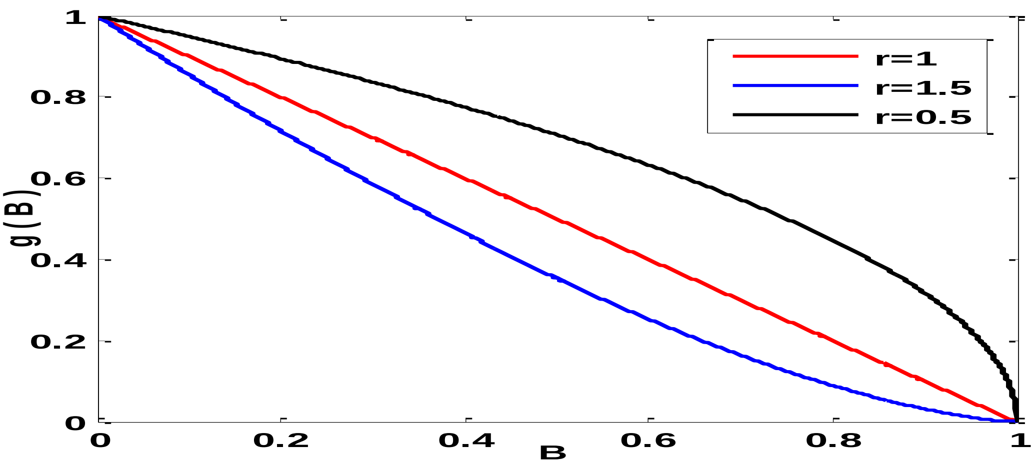
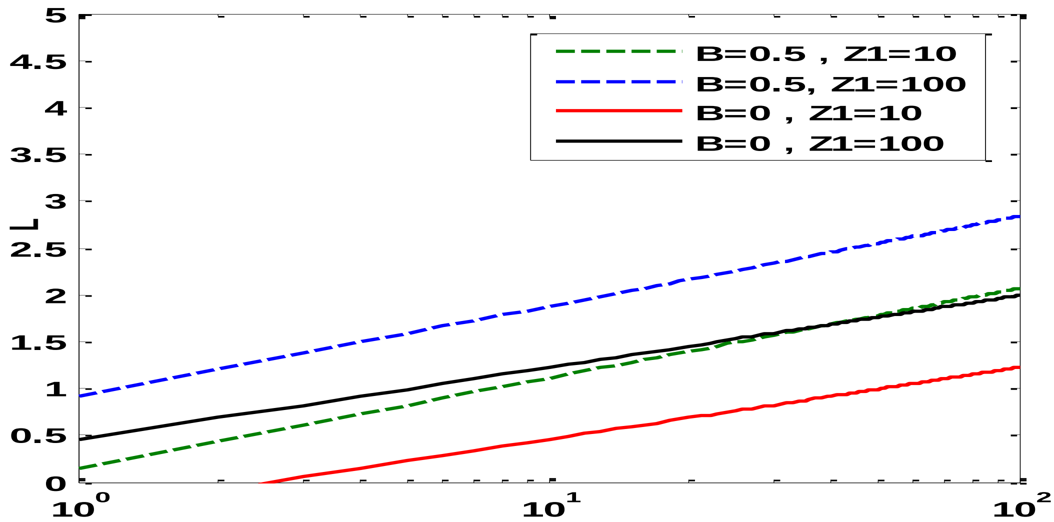
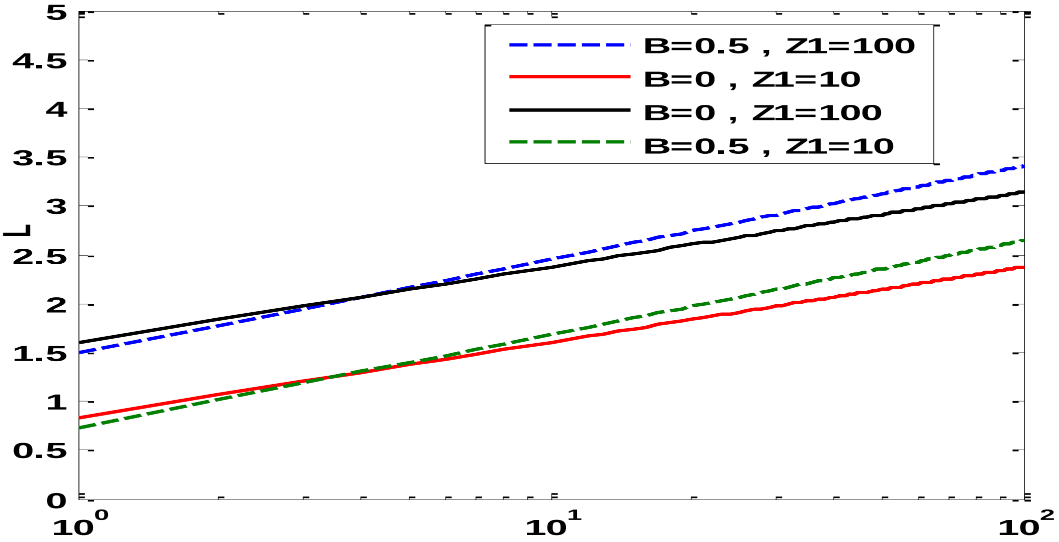


Fig.(2)Ratio $a/a_{TF} = g(\beta)$ versus charge stat

Figure (3) shows the results of Bethe stopping number which are obtained from eq.(24) as a function of Bohr parameter ξ with values of $(\beta = 0,\ Z_1 = 100)$, $(\beta = 0,\ Z_1 = 10)$, $(\beta = 0.5,\ Z_1 = 100)$ and $(\beta = 0.5,\ Z_1 = 10)$ for (a) $s = 10$ and (b) $s = 1$. It is seen that in each curve, the stopping number increases with increasing ξ. The figure also indicates a greater sensitivity to the atomic number of projectile Z_1 than to the fractional charge β and the stopping number increases significantly with increasing Z_1 at constant ξ and β and this dependence confirms the transition from Bohr to Bethe behavior moves rapidly to greater values of ξ with increasing Z_1 at all values of β. From a and b, we note that the stopping number is sensitive to the velocity parameter at s =1, while for s =10 a weaker dependence is found. The variation of velocity parameter s with the charge state β is apparent, but the variation with ξ is less evident.



(a) $s = 10\xi$

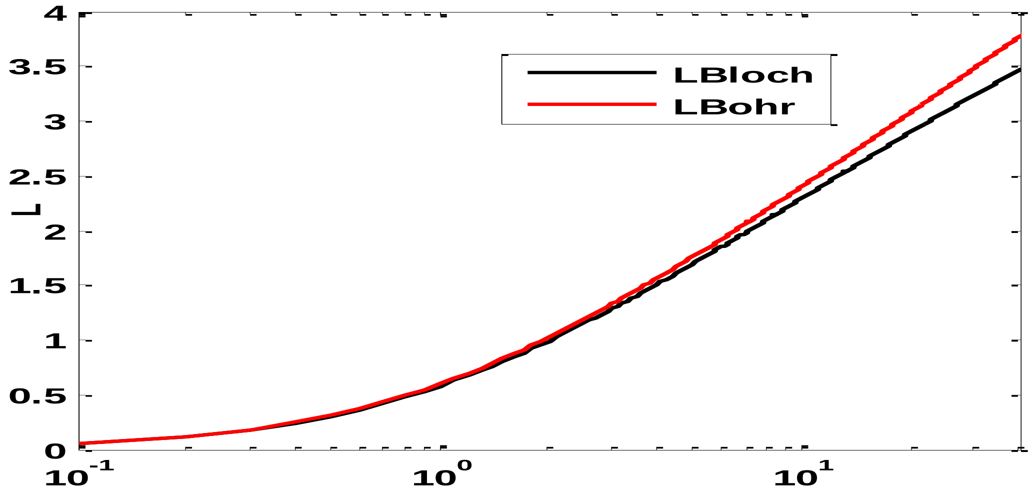


(b) $s = 1$

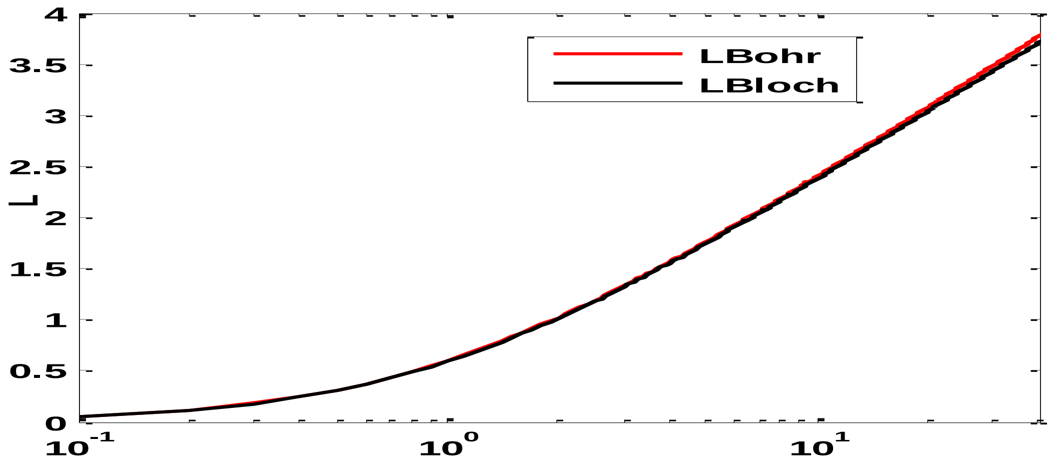
Fig.(3) Bethe stopping number plotted in variables appropriate to Bohr scaling

Figure (4) shows the results of Bloch stopping number that are calculated from modified Bloch formula eq.(39) and the Bohr stopping number that are calculated from the total stopping number eq.(6) as a function of Bohr parameter ξ with (a) $\alpha = 2$ and (b) $\alpha = 4$. It is seen that the stopping number of modified Bloch formula coincide with that of

Bohr formula at low velocity because the inverse Bloch corrections at low velocity approaching zero and $L_{Bloch} = L_{Bohr}$ while a difference occurs at high velocity which, in essence, is the difference between the Bethe and unmodified Bohr formula. The difference increases with decreasing α (for small values of K)−i.e., when the Bloch formula approaches the Bethe while it diminishes rapidly for larger values of α (for large values of K). Already for $\alpha = 4$ the difference becomes invisible.



(a) $\alpha = 2\xi$



(b) $\alpha = 4\xi$

Fig.(4) Stopping number from modified Bloch formula compared to the result from the modified Bohr formula

Figure (5) shows the results of Bethe stopping number (Bethe logarithm) that are calculated from eq.(21) and modified Bloch stopping number that are calculated from eq.(47) as a function of ($x = 2mv^2/\hbar\,\omega$). From the figure, it is seen that the results of Bethe and Bloch stopping number are in perfect agreement at the high speed limit because the Bloch correction ΔL_{Bloch} goes to zero hence, $L_{Bloch} = L_{Bethe}$ at high speed while there is a difference between the results at low speed because the Bloch formula approaches the Bohr limit. This difference is ascribed to the corrected form of modified Bloch formula in order to avoiding the negative stopping number that occurs in the Bethe formula at low velocity $2mv^2 < \hbar\,\omega$.

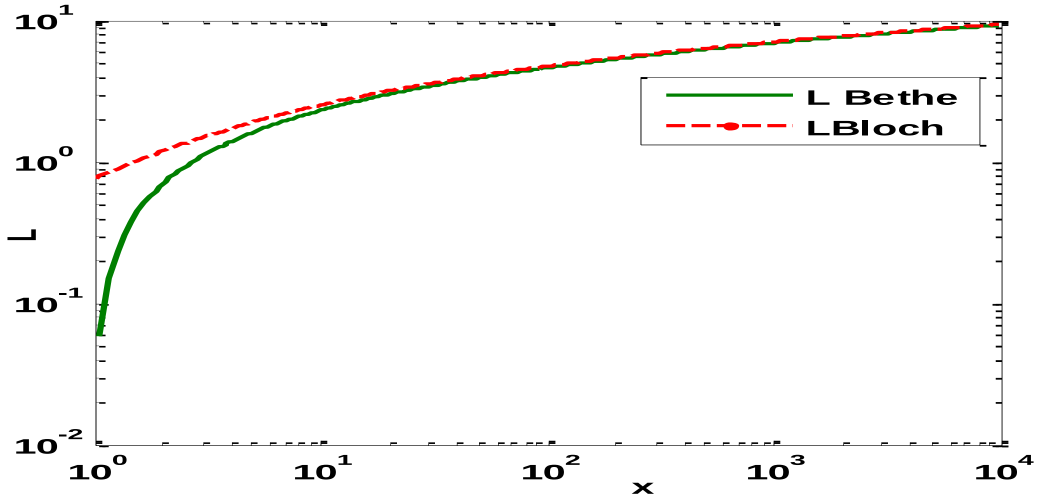
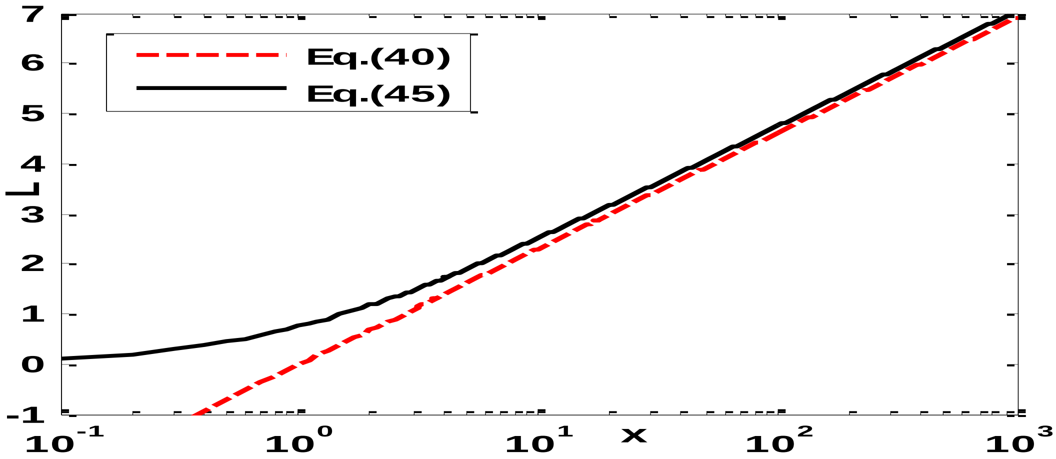
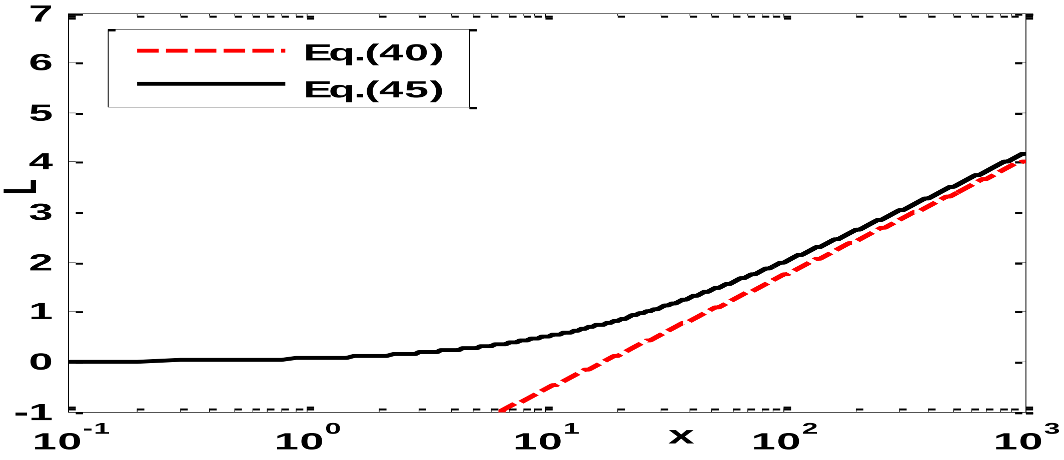


Fig.(5)Comparison between modified Bloch formula and Bethe formula

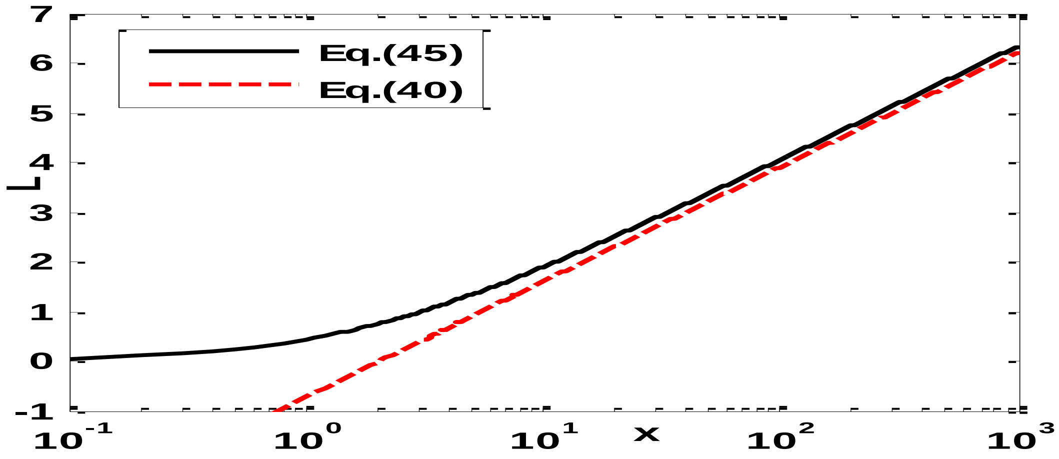
Figure (6) shows the results of Bloch stopping number which are obtained from eqs.(40 and 45) as a function of x with different values of inverse Sommerfeld parameter$(1/\eta = v/Z_1\,v_0)$ (a)$1/\eta$=10, (b) $1/\eta$=0.1 and (c) $1/\eta$=1. From the figure, it is noted that the results of stopping number from the two equations are coincident at high speed with the difference in values of Somerfield parameter for(a)$1/\eta$=10 the Bethe regime dominates, for (b)$1/\eta$=0.1 the Bohr regime dominates and (c)$1/\eta$=1 the transition regime dominates. It is also seen that when $2mv^2/\hbar\,w \ll 1$, L_{Bloch} from eq.(45) approaches zero and when $2mv^2/\hbar\,w \to 1$, L_{Bloch} from eq.(45) takes a positive values but when $2mv^2/\hbar\,w \ll 1$, L_{Bloch} from eq.(40) takes a negative values and when $2mv^2/\hbar\,w \to 1$, L_{Bloch} from eq.(40) approaches zero. The Bloch stooping number depends on the Sommerfeld parameter η and proportional parallel with the $1/\eta$.



(a) $1/\eta$*=10*



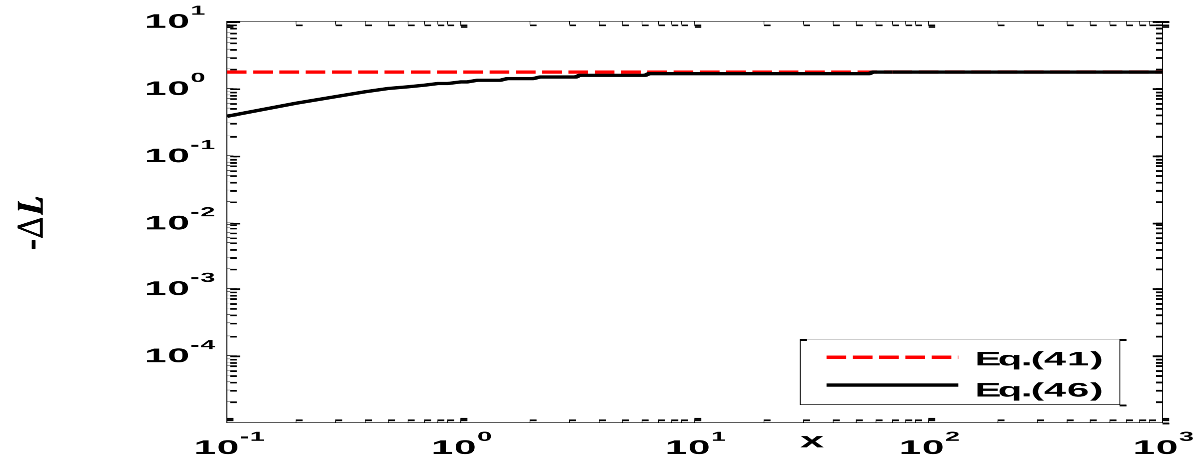
(b) 1/η= 0.1



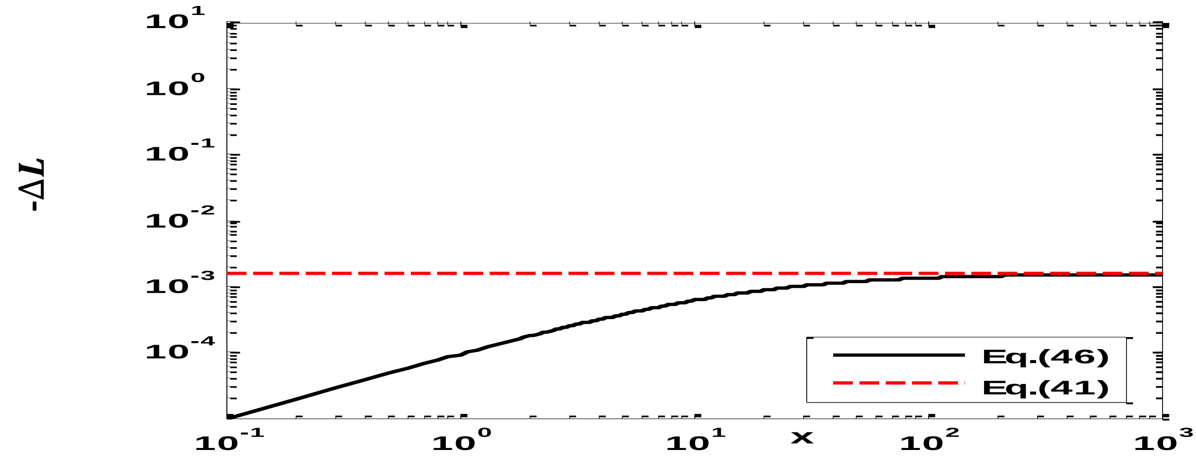
(c) 1/η= 1

Fig.(6)Approximation to Bloch stopping number

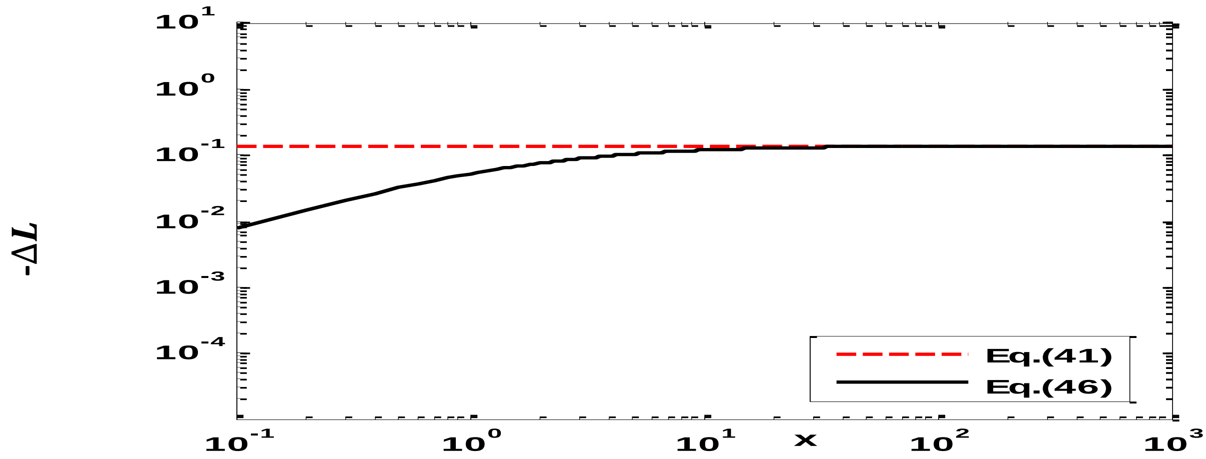
Figure (7) shows the results of inverse Bloch correction which are evaluated from eqs.(41, 46) as a function of x with different values of inverse Sommerfeld parameter ($1/\eta = v/Z_1 v_0$) (a) $1/\eta$ =10, (b) $1/\eta$ =0.1 and (c) $1/\eta$ =1. The results of two equations are coincident at high velocities while there is a difference in results at low velocities. The coincidence increases and the difference decreases with increasing the value of $1/\eta$. The values of inverse Bloch correction obtained from eq.(41) remain constant and not change but the results from eq.(46) change with decreasing the value of $1/\eta$.



(a) $1/\eta$*=10*



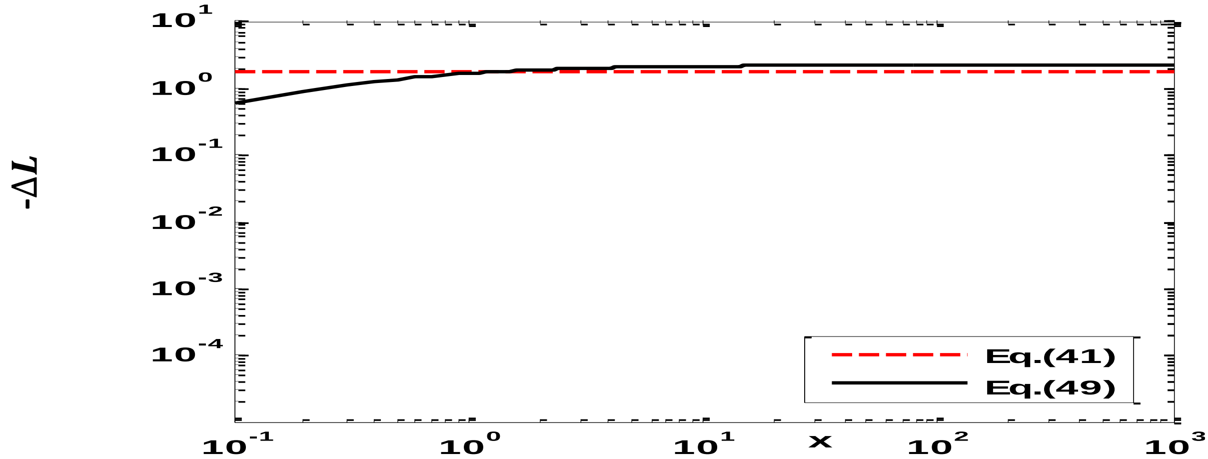
(b)$1/\eta$*=0.1*



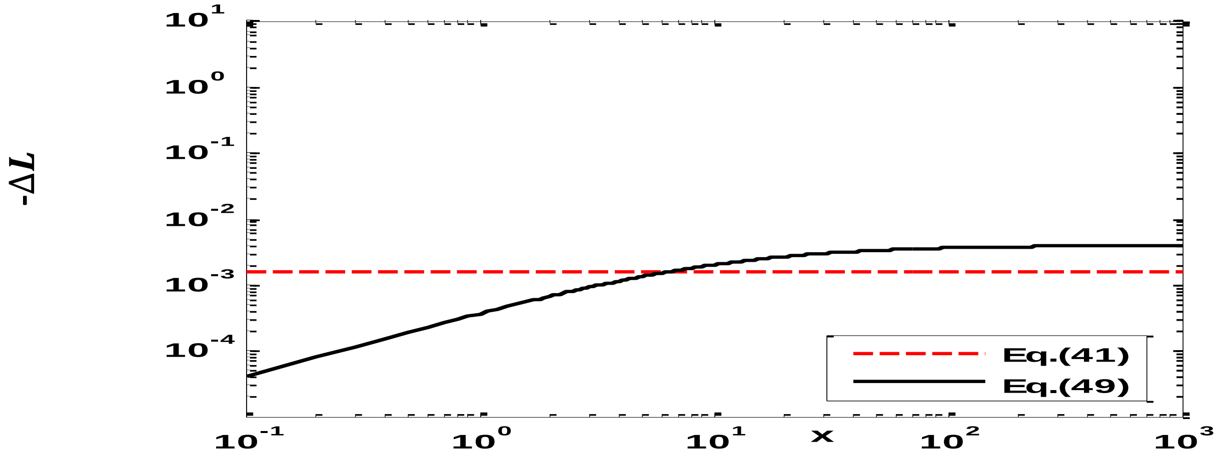
(c) $1/\eta$*=1*

Fig.(7) Inverse Bloch correction

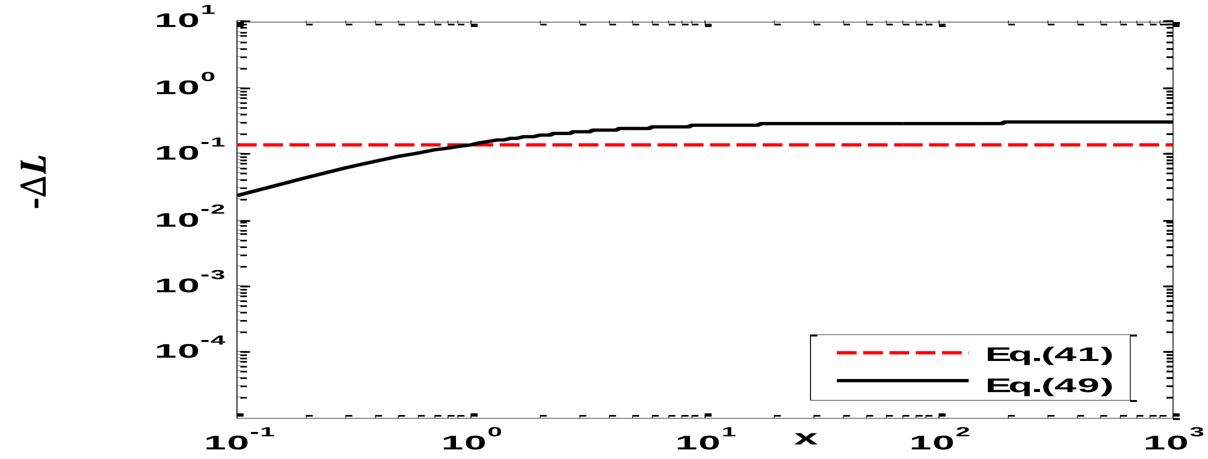
Figure (8) shows the results of inverse Bloch correction which are calculated from eqs.(41,49) as a function of x with different values of inverse Sommerfeld parameter $(1/\eta = v/Z_1 v_0)$ (a)$1/\eta$=10, (b) $1/\eta$=0.1 and (c) $1/\eta$=1. From the figure, it is seen that there is a difference in results from the two equations at high and low velocities. The difference increases with decreasing the value of $1/\eta$. From the figures (7 and 8), the values of inverse Bloch correction depend on the Sommerfeld parameter η and proportional parallel with the $1/\eta$.



(a)1/*η*=10



(*b*) 1/*η*=0.1



***(c)* 1/*η*=1**

Fig.(8)Inverse Bloch correction

4. Conclusions

Studying stopping power and understanding slowing down of charged particles in target has a continuous interest in physics and other fields because of its broad application in radiation damage, radiology, nuclear physics and ion implantation.

The stopping power of particles at low charge and high speed like electrons and protons precisely is evaluated by Bethe theory because its validity is restricted for charged particles with low Z_1 (light) and high v (fast) only and treats target – projectile interaction by quantal perturbation theory to lowest order but the precision decline with increasing projectile charge and decreasing the speed of it, therefore it must be necessary to begin at the other end (classical end), i.e. Bohr theory and it is superior to Bethe's theory when Bohr kappa criterion $K = \frac{2Z_1 v_0}{v} > 1$. The influence of projectile electrons is taken into account and these electrons are the basis of screening of the Coulomb interaction. The importance of projectile screening hinges on the velocity parameter s which depends equally on the target and projectile.

The Bohr formula for the stopping number is governed by a logarithm and it becomes negative at low projectile speed for $Cmv^3 < Z_1 e^2 \omega$. The negative values of stopping number are caused by asymptotic expansion of Bessel function. Hence, this can be repaired by need for various corrections to the Bohr theory at low velocity. The modified Bohr formula was found in order to reproduce the qualitative behavior of the Bohr stopping number without addition any correction (shell and Barkas corrections) at low velocity which are necessary to the Bohr formula. In the total stopping number, the contribution from close collisions is more than that from distant collisions because collisions dominate in high and low projectile speed but distant collision is in low projectile speed only.

Bloch showed that Bohr's harmonic oscillator approach might be converted into a quantitative tool by incorporation of an inverse Bloch correction. The Bloch function of stopping number reduces to Bethe formula $L_{Bloch} = L_{Bethe}$ at high projectile speed but at low projectile speed, $\Delta L_{invBloch}$ goes to zero and hence $L_{Bloch} = L_{Bohr}$. The inverse Bloch correction depends on the Sommerfeld parameter η and proportional inversely with the η .

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الانتقال من النظرية الكلاسيكية إلى الكمية لحساب قدرة الإيقاف للجسيمات المشحونة

سناء ثامر كاظم

الخلاصة:-

في هذا البحث تم وصف أكثر التصحيحات أهمية فيما يخص ا لصيغة القياسية للطاقة المفقودة للجسيمات المشحونة وكذلك قمنا بمناقشة تلك الصيغة مع بعض الملاحظات وذلك عن طريق تضمين تصحيح بلوخ العكسي إلى حسابات قدرة الإيقاف. إن تصحيح بلوخ العكسي يوسع المدى فيما يخص تحقيق نظرية بور الكلاسيكية وبيتا الكمية. فنموذج بور يتعامل مع تصادم الجسيمات المشحونة بالمادة الهدف على أساس عدد من التفاعلات بين الجسيم الساقط المشحون والكترونات الهدف المقيدة توافقيا بينما في نظرية بيتا فان جميع حسابات الطاقة المفقودة للجسيمات المشحونة السريعة تستند على تقريب بورن الأول الذي فيه يعتبر كل النظام الفيزيائي مكمما. ان جميع النتائج قد تم الحصول عليها عن طريق برمجه المعادلات ضمن برنامج *الماتلاب* .